Forecasting Attrition Volume

A Methodological Development

Manchun Fang
Attrition and Retention
Personnel Generation Research

Stephen Okazawa
Land Force Operational Team
Centre of Operational Research and Analysis

Defence R&D Canada
Director General Military Personnel Research & Analysis

Chief Military Personnel

National Defence   Défense nationale

Forecasting Attrition Volume

A Methodological Development

Manchun Fang
Attrition and Retention
Personnel Generation Research

Stephen Okazawa
Land Force Operational Team
Centre of Operational Research and Analysis

Principal Author

(Original signed by)

Manchun Fang, MSc

Approved by

(Original signed by)

Douglas Pelchat, BA

Section Head – Personnel Generation Research

Approved for release by

(Original signed by)

Kelly Farley, PhD

Chief Scientist – Director General Military Personnel Research and Analysis

The opinions expressed in this paper are those of the authors and should not be interpreted as the official position of the Canadian Forces, nor of the Department of National Defence.

Abstract

Accurate attrition forecasting is crucial for properly planning the recruitment and training of Canadian Forces (CF) members and maintaining the CF strength as well as for managing the CF budget.

This report documents a methodological development in forecasting attrition volume; new procedures for forecasting attrition based on years of service (YOS) have been proposed. The report provides a discussion regarding the rational for the new procedures and explanations why the predictions based on new procedures better reflect CF attrition behaviour. In the end, the new procedures were validated and compared with previous procedures using the real CF personnel data.

The results from the new procedures showed a strong agreement between forecast and actual attrition. Compared with the previous forecasting method, the new procedures deliver predictions that better reflect CF attrition behaviour at many levels.

The report recommends using the proposed procedures for forecasting CF attrition. These procedures can be applied to attrition analyses at many levels within the CF, for example, non-commissioned members (NCMs) and officers (OFFs), Army, Navy and Air Force, different military branches and different military occupations, etc.

This report is targeted at the analysts within Defence Research and Development Canada. This work will equip them with a better approach for forecasting CF attrition, and improve the consistency and transparency of attrition analyses across different research groups. Given that forecasting attrition is so important for a number of relevant human resources initiatives, such as effective recruitment, promotion, planning, and budget management, this work will have a positive impact in all of these areas.

Résumé

Prévoir l'attrition est essentiel pour planifier correctement le recrutement et l'instruction des membres des Forces canadiennes (FC), de maintenir le potentiel des FC et d'en gérer le budget.

Le présent rapport expose une nouvelle méthodologie de prévision de l'attrition; de nouvelles méthodes pour prévoir l'attrition en fonction du nombre d'années de service (AS) y sont proposées. Le rapport présente une analyse du bien-fondé des nouvelles méthodes et explique pourquoi les prévisions fondées sur ces nouvelles méthodes expriment mieux l'attrition dans les FC. Enfin, les nouvelles méthodes sont validées et comparées aux anciennes au moyen de données réelles sur l'effectif des FC.

Les résultats obtenus grâce aux nouvelles méthodes ont révélé une grande concordance entre les prévisions de l'attrition et les données réelles en la matière. Par comparaison avec l'ancienne méthodologie, les nouvelles méthodes permettent d'établir des prévisions qui illustrent mieux l'attrition aux différents échelons des FC.

Les auteurs du rapport recommandent l'utilisation des méthodes proposées pour prévoir l'attrition dans les FC. Ces méthodes peuvent être appliquées à l'analyse de l'attrition aux différents échelons des FC, par exemple chez les militaires du rang (MR) et les officiers (offr), dans l'Armée de terre, les Forces maritimes et la Force aérienne, ainsi que dans d'autres branches militaires et emplois militaires, etc.

Le présent rapport est destiné aux analystes de Recherche et développement pour la défense Canada (RDDC). Il leur permettra de mieux prévoir l'attrition dans les FC et améliorera la cohérence et la transparence des analyses de l'attrition des divers groupes visés par la recherche. La prévision de l'attrition étant très importante pour nombre d'interventions en matière de ressources humaines, comme le recrutement, la promotion, la planification et la préparation des budgets, ce rapport aura des répercussions positives sur tous ces plans.

Executive summary

Forecasting Attrition Volume: A Methodological Development

Manchun Fang; Stephen Okazawa; DGMPRA TM 2009-025; Defence R&D Canada – DGMPRA; December 2009.

Attrition is an important factor to consider in the context of Canadian Forces (CF) expansion efforts. High personnel turnover in the CF has serious consequences, and CF management is paying more and more attention to attrition and retention issues at various levels. Knowledge of CF attrition and its underlying causes is crucial to effective personnel management.

Director General Military Personnel Research and Analysis (DGMPRA) has long history of research on reporting and forecasting attrition. Recently a review of traditional methods for reporting and forecasting attrition was conducted, with the goal to provide a rigorous methodology and develop standardized procedures for CF attrition analyses in order to better support CF personnel management.

This report documents the latest methodological development in forecasting attrition volume. First, new procedures for forecasting Years of Service (YOS) - based attrition are presented. Second, the theoretical reasons explaining why the new procedures are appropriate and should be used for forecasting Regular Force (RegF) attrition are discussed. Then, a visual comparison and a statistical test compare the difference between the actual attrition volumes and the forecast attrition volumes using the new procedures. The results show a good fit between the attrition model built by the new procedures and the actual CF attrition behaviour.

For the purpose of an objective comparison, the new procedures were compared to previous methods using data from a number of different historical periods. The results demonstrated that generally the new procedures produce more accurate predictions of YOS-based attrition, which better reflect CF attrition behaviour than the previous procedures.

The documentation of this work is a step towards improving the consistency and transparency of attrition analyses carried out by different research groups. Given that forecasting attrition is very important to a number of relevant human resources initiatives, such as recruitment, promotion, planning and budget management, this work will have a positive impact in all of these areas.

The new procedures are recommended for forecasting CF-wide attrition, and where appropriate, for attrition forecasting of different levels within the CF, such as NCM and Officer, Army, Navy and Air Force, different military branches and different military occupations. The new procedures are currently used in the Personnel Generation Research Section within DGMPRA for various attrition applications.

Sommaire

Forecasting Attrition Volume: A Methodological Development

Manchun Fang; Stephen Okazawa; DGMPRA TM 2009-025; R & D pour la défense Canada – DRASPM; Décembre 2009.

L'attrition est un important facteur à prendre en considération dans le contexte des efforts d'expansion des Forces canadiennes (FC). Un taux de roulement élevé dans les FC entraîne des conséquences graves, et l'administration des Forces canadiennes accorde de plus en plus d'attention à l'attrition et au maintien de l'effectif à plusieurs égards. La connaissance du phénomène de l'attrition dans les FC et de ses causes sous-jacentes est essentielle pour assurer une gestion efficace de l'effectif.

Depuis déjà un bon moment, la Direction générale – Recherche et analyse (Personnel militaire) (DGRAPM) s'intéresse aux façons de rendre compte de l'attrition et de la prévoir. Récemment, un examen des méthodes traditionnelles de rapport et de prévision de l'attrition a été effectué dans le but de trouver une méthodologie rigoureuse et de mettre en place des méthodes normalisées pour l'analyse de l'attrition dans les FC de manière à mieux soutenir la gestion de l'effectif des FC.

Le présent rapport expose la plus récente méthodologie en matière de prévision de l'attrition. Tout d'abord, de nouvelles méthodes pour prévoir l'attrition en fonction des années de service (AS) y sont présentées, suivies des justifications du bien-fondé des nouvelles méthodes et des raisons pour lesquelles elles devraient être utilisées pour prévoir l'attrition dans la Force régulière (F rég). Une comparaison visuelle et un test statistique mettent ensuite en parallèle les différences entre l'attrition réelle et l'attrition prévue au moyen des nouvelles méthodes. Les résultats font état d'une bonne concordance entre les prévisions de l'attrition établies à l'aide des nouvelles méthodes et l'attrition réelle observée dans les FC.

Pour garantir une comparaison objective, les nouvelles méthodes ont été confrontées aux anciennes à l'aide de données sur un certain nombre de périodes passées. De façon générale, les résultats ont montré que les nouvelles méthodes permettaient de prévoir avec plus d'exactitude l'attrition en fonction des années de service, ce facteur étant plus révélateur du phénomène de l'attrition que les anciennes méthodes.

Le présent rapport a pour objet d'améliorer la cohérence et la transparence de l'analyse de l'attrition visant différents groupes. La prévision de l'attrition étant très importante pour nombre d'interventions en matière de ressources humaines, comme le recrutement, la promotion, la planification et la préparation des budgets, ce rapport aura des répercussions positives sur tous ces plans.

Il est par conséquent recommandé d'appliquer les nouvelles méthodes pour prévoir l'attrition dans l'ensemble des FC, et le cas échéant, pour prévoir l'attrition aux différents échelons des FC, notamment chez les militaires du rang (MR) et les officiers (offr), dans l'Armée de terre, les Forces maritimes et la Force aérienne, ainsi que dans d'autres branches militaires et emplois militaires. Les nouvelles méthodes sont actuellement utilisées à la Direction générale – Recherche et analyse (Personnel militaire) (DGRAPM) dans divers contextes liés à l'attrition.

Table of contents

List of figures

List of tables

Acknowledgements

1   Background

Attrition is essential to the management of the continuous flow of new recruits through to the most senior officers and non-commissioned members (NCMs). However, high personnel turnover, or attrition, in the Canadian Forces (CF) has serious consequences, and any amount of higher-than-expected attrition in an under-strength or very small occupation will be problematic.

Forecasting of attrition is vital to CF Human Resources planning. Accurate knowledge of attrition is crucial to properly planning the recruitment and training of CF members as well as for managing the personnel budget. In short, the better the forecast, the more effectively CF personnel can be managed.

Director General Military Personnel Research and Analysis (DGMPRA) has a long history of methodological research relating to attrition and producing attrition analyses for military clients [1-8]. Recently the Personnel Generation Research Section in DGMPRA conducted a review of traditional methods of reporting and forecasting attrition. The goal was to provide a more rigorous methodology and standardized procedures for reporting and forecasting CF attrition. This would support CF personnel management and improve the consistency of methodologies used within the department. In 2007 Okazawa [1] published *Measuring Attrition and Forecasting Attrition Volume* and provided a new set of equations for attrition analysis.

Attrition is a function of many factors, such as occupation, demographic profile, economic factors, policies, etc. In particular, attrition is strongly related to Years of Service (YOS) because most attrition occurs at or shortly after exit gates between engagements, and most engagements have a prescribed duration measured in YOS (see Figure 1). For example, at 20 YOS (under the previous Terms of Service[1]), CF members have completed their Intermediate Engagement (IE). At this point, the CF experiences a high attrition rate because many of its members choose to retire rather than serve on an Indefinite Period of Service (IPS) [1]. Another spike in the CF attrition profile occurs at the 0 YOS point. The CF experiences much higher attrition with members with 0 YOS[2] than any other YOS groups prior to the 20 YOS point, i.e., before the pensionable service point. The information on attrition by YOS is critical to CF personnel management. Thus both total annual attrition volume and attrition by YOS are frequently reported and included in forecasts.

---

[1]   The new Terms of Service were introduced in 2005, which changed the intermediate engagement from 20 YOS to 25 YOS.

[2]   0 YOS is the period from the first day of the new members' enrolment to their one year anniversaries.
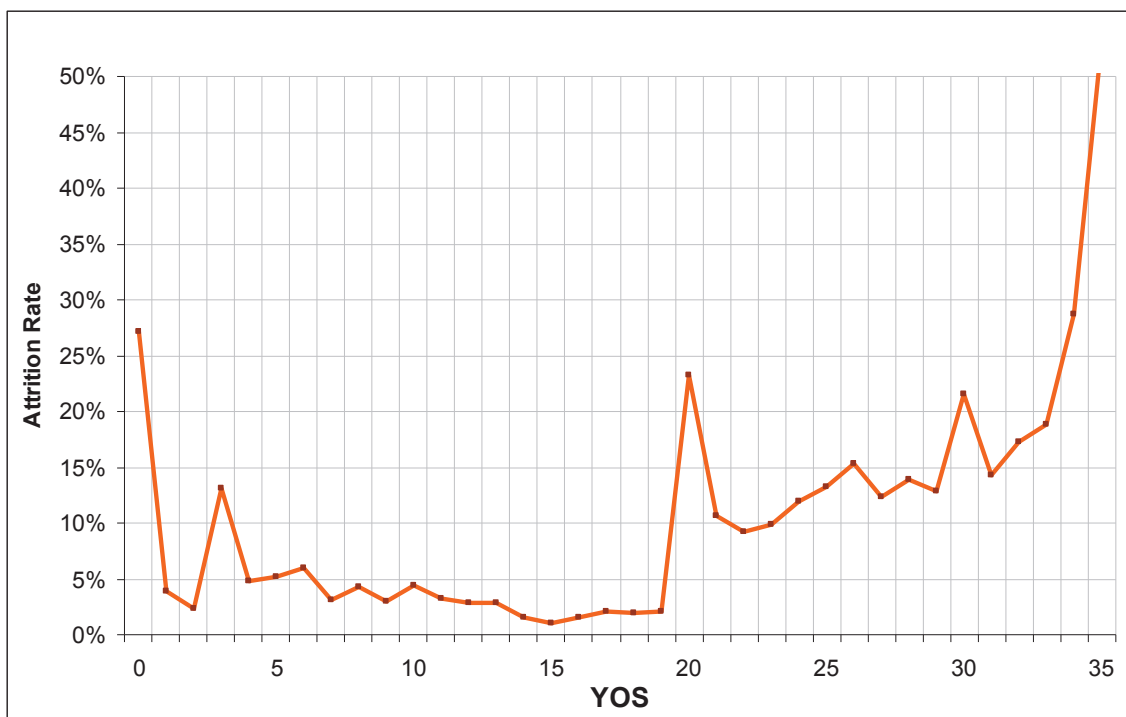
Figure 1: A Typical Regular Force (Reg F) Attrition Profile

Okazawa's equations provide a method to forecast the number of releases of individuals with a given YOS where YOS is measured at the beginning of the next year (fiscal or calendar). Missing from this original paper was a specific method to forecast the number of releases of individuals with a given YOS where YOS is measured at the time of their release. This is a more useful attrition statistic than the one measured at the beginning of the next year. The difference seems subtle, but the previous method does not exactly capture attrition behaviour at each YOS. The deviation will be larger at critical YOS points, e.g., 0 YOS, and YOS points at engagement gates (e.g. 20 YOS).

This report presents a methodological extension to the forecasting techniques described in Okazawa 2007 [1] to addresses the YOS related attrition behaviour. The new procedures for forecasting attrition are described and discussed in detail in the following sections. First, new procedures for forecasting YOS-based attrition are presented in Section 2. Next, the theoretical reasons why the new procedures are appropriate and should be used for forecasting RegF attrition are discussed in Section 3. Then, empirical evidence of the performance of the new procedures are presented in Section 4.

The new approach for forecasting CF attrition described herein is currently in use by the Attrition/Retention and the Workforce Modelling Teams in the Personnel Generation Research Section of DGMPRA, and has been used for various CF attrition studies and applications, e.g., attrition analyses for Annual Military Occupational Reviews, and the Annual Report on Regular Force Attrition [9]. This work will help analysts provide more accurate attrition

projections to senior leadership, including Career Field and Occupation Authorities. Given that forecasting attrition is vitally important for a number of relevant CF Human Resources initiatives, such as recruitment, promotion, planning and budget management, this work will have a positive impact in all these areas. Documenting this work will also improve the transparency and consistency of attrition analyses across different research groups within Defence Research and Development Canada (DRDC).

2   Method

Attrition volume is defined as the number of members who are released from the Regular Force in a given year. Attrition rate is the proportion of individuals in a population at a point in time who will attrite within the year.

The following methodologies can be applied to attrition forecasting for the future calendar year or fiscal year. Therefore, the starting point of the year can be January 1st or April 1st respectively.

2.1   Notation

Listed below is the notation used in the following sections. Most of the notation is the same as that from the previous report [1], for consistency.

α	Yearly attrition rate
$a_m[n]$	Number of releases in year n-1, where the released members have m YOS, where YOS is measured on Jan 1st /Apr 1st of year n
$a'_m[n]$	Number of releases in year n-1, where the released members have m YOS measured at the time of release
$A[n]$	Predicted attrition volume for year n-1, for all YOS
$A_m[n]$	Predicted attrition volume for year n-1, for members with m YOS measured on Jan 1st/ Apr 1st of year n
$P[n]$	Population on Jan 1st/ Apr 1st of year n
$P_m[n]$	Population with m YOS on Jan 1st/ Apr 1st of year n
$r[n]$	Total recruitment occurring in year n
$T_m[n]$	Transfers/Recruits in year n-1 with m YOS, where YOS is measured on Jan 1st/ Apr 1st of year n
$WAAR(\alpha_m)$	YOS based Weighted Average Attrition Rate
$WAAR(\alpha_{m-1,m})$	Net Weighted Average Attrition Rate

2.2 Forecasting Attrition by YOS

As noted earlier, attrition is strongly related to YOS. Therefore, forecasting attrition at each YOS, especially forecasting attrition at each YOS where YOS is measured as of the release date, is required.

Table 1 lists the new procedures for predicting attrition by YOS, where YOS is measured at their release date. Table 2 lists the previous procedure for forecasting attrition by YOS, where YOS is measured at the end of the year. Note that steps 1, 2 and 4 differ between the two procedures. Step 3 is the same in both procedures.

In order to forecast attrition, historical attrition rates by YOS need to be calculated. The attrition rates can be calculated based on historical data from a single year or from several previous years.[3] If data from multiple previous years are used, the Weighted Average or Least Squares approach can be used for getting an overall estimate for the attrition rate. In this report, the Weighted Average approach is used in both the new and old procedures presented in Tables 1 and 2. The new procedures using the Least Squares approach are listed in Annex A.

The derivations of these formulae are not presented in this report. Some of the derivations can be found in [1]. The focus of this report is to document the revised methodology for forecasting attrition, explain the rationale for the changes, provide a comparison of forecasts between the original and this revised version, and provide an overview on choosing the right formula and using the formula in the right way.

---

[3] The amount of history to use is discussed in Fang and Bender (2008), reference [8].

Table 1: The New Procedures for Predicting Attrition by YOS

Steps	New Procedures
1	Obtain $a'_m[n]$, $a_m[n]$, $P_m[n]$, and $T_m[n]$ from historical databases on RegF personnel; and obtain the targeted number of recruits $T_m[n]$ from the Strategic Intake Plan (SIP), which is issued by Director Personnel Generation Requirements (DPGR).
2	Calculate the YOS-based Weighted Average Attrition Rate WAAR (α_m) based on $a'_m[n]$ using the following formulae from [1]. When m>0, $$WAAR(\alpha_m) = \frac{\sum_{n=1}^{N} a'_m[n]}{\sum_{n=1}^{N}\left(\frac{1}{2}P_{m-1}[n-1] + \frac{1}{2}P_m[n-1] + \frac{1}{3}T_m[n] + \frac{1}{6}T_{m+1}[n]\right)} \qquad (1)$$ When m=0 (i.e., YOS=0), $$WAAR(\alpha_0) = \frac{\sum_{n=1}^{N} a'_0[n]}{\sum_{n=1}^{N}\left(\frac{1}{2}P_0[n-1] + \frac{1}{2}T_0[n] + \frac{1}{6}T_1[n]\right)} \qquad (2)$$
3	Forecast $P_m[n]$ by using the following formula: $$P_m[n] = \left(1 - WAAR(\alpha_{m-1,m})\right)P_{m-1}[n-1] + \left(1 - \frac{1}{2}WAAR(\alpha_{m-1,m})\right)T_m[n] \qquad (3)$$ When m>0, $$WAAR(\alpha_{m-1,m}) = \frac{\sum_{n=1}^{N} a_m[n]}{\sum_{n=1}^{N}\left(P_{m-1}[n-1] + \frac{1}{2}T_m[n]\right)} \qquad (4)$$ Where $T_m[n]$ is the number of the transfers (including new recruits) in future year n-1, which can be estimated by considering both the targeted number of recruits from the SIP and the historical number and YOS distribution of occupational transfers.

DGMPRA TM 2009-025

	When m=0, $$WAAR(\alpha_0) = \frac{\displaystyle\sum_{n=1}^{N} a_0[n]}{\displaystyle\sum_{n=1}^{N}\left(\frac{1}{2}T_0[n]\right)} \qquad (5)$$ and $T_0[n]$ is equal to number of recruits for the year n.
4	Forecast attrition volume in year n-1 for personnel with m YOS. When m>0, $$A_m[n] = WAAR(\alpha_m) \times \left(\frac{1}{2}P_{m-1}[n-1] + \frac{1}{2}P_m[n-1] + \frac{1}{3}T_m[n] + \frac{1}{6}T_{m+1}[n]\right) \qquad (6)$$ When m=0, the following formula from [1] can be used: $$A_0[n] = WAAR(\alpha_0) \times \left(\frac{1}{2}P_0[n-1] + \frac{1}{2}T_0[n] + \frac{1}{6}T_1[n]\right) \qquad (7)$$ Note that the $WAAR(\alpha_m)$ used in formula (6) and (7) is calculated from $a'_m[n]$ and not from $a_m[n]$.

Table 2: Previous Procedures for Forecasting Attrition by YOS

Steps	Previous Procedures
1	Obtain $a_m[n]$, $P_m[n]$, and $T_m[n]$ from historical databases on RegF personnel; and obtain the targeted number of recruits $T_m[n]$ from the SIP.
2	Calculate net YOS based attrition rates, that is, Weighted Average net YOS-based attrition rate WAAR ($\alpha_{m-1,m}$) based on $a_m[n]$ by using the following formulae. When m>0, $$WAAR(\alpha_{m-1,m}) = \frac{\displaystyle\sum_{n=1}^{N} a_m[n]}{\displaystyle\sum_{n=1}^{N}\left(P_{m-1}[n-1] + \frac{1}{2}T_m[n]\right)} \qquad (4)$$ When m=0, $$WAAR(\alpha_0) = \frac{\displaystyle\sum_{n=1}^{N} a_0[n]}{\displaystyle\sum_{n=1}^{N}\left(\frac{1}{2}T_0[n]\right)} \qquad (5)$$ where $T_0[n]$ is equal to the number of recruits for the year n-1.
3	Forecast $P_m[n]$ (that is the population with m YOS on Jan 1st/Apr 1st of year n) by using following formula. $$P_m[n] = \left(1 - WAAR(\alpha_{m-1,m})\right)P_{m-1}[n-1] + \left(1 - \frac{1}{2}WAAR(\alpha_{m-1,m})\right)T_m[n] \qquad (3)$$
4	Forecast attrition with m YOS at year n. When m>0, $$A_m[n] = P_{m-1}[n-1] - P_m[n] + T_m[n] \qquad (8)$$ When m=0 (i.e., YOS=0), $$A_0[n] = -P_0[n] + T_0[n] \qquad (9)$$

3   Method

The following section provides a detailed description of the new method for forecasting attrition by YOS and forecasting the total annual attrition volume.

3.1   Forecasting Attrition by YOS

3.1.1   Major Differences between the Two Procedures

The major differences between the previous and the new forecasting procedures are as follows:

a.   First, the attrition rates used for forecasting are calculated using different formulae. With the new procedures the attrition rate is calculated based on $a'_m[n]$, (i.e., the number of releases in year n-1 with m YOS measured at the time of release), while in the previous procedure, the attrition rate was calculated based on $a_m[n]$ (i.e., the number of releases in year n-1 with m YOS measured on the beginning of year n). Mathematically, the new procedure uses the $WAAR(\alpha_m)$:

$$WAAR(\alpha_m) = \frac{\sum_{n=1}^{N} a'_m[n]}{\sum_{n=1}^{N}\left(\frac{1}{2}P_{m-1}[n-1]+\frac{1}{2}P_m[n-1]+\frac{1}{3}T_m[n]+\frac{1}{6}T_{m+1}[n]\right)} \qquad (1)$$

while the previous procedure uses the $WAAR(\alpha_{m-1,m})$:

$$WAAR(\alpha_{m-1,m}) = \frac{\sum_{n=1}^{N} a_m[n]}{\sum_{n=1}^{N}\left(P_{m-1}[n-1]+\frac{1}{2}T_m[n]\right)} \qquad (4)$$

b.   The second major difference is that the new procedure uses equation (6) from Table 1 for forecasting attrition by YOS:

$$A_m[n] = WAAR(\alpha_m) \times \left(\frac{1}{2}P_{m-1}[n-1]+\frac{1}{2}P_m[n-1]+\frac{1}{3}T_m[n]+\frac{1}{6}T_{m+1}[n]\right) \qquad (6)$$

the previous procedure instead uses formula (8) from Table 2:

$$A_m[n] = P_{m-1}[n-1] - P_m[n] + T_m[n] \qquad (8)$$

As a result, the new procedures enable us to predict true attrition by YOS where YOS is *measured at the release date*.

3.1.2 Formulation of the New Methodology

There are three ways one may think of to forecast attrition (two of which are invalid).

3.1.2.1 Two Invalid Ways

1. First, given the following basic equation relating population, transfers and attrition:

$$P[n] = P[n-1] + T[n] - A[n] \qquad (10)$$

The meaning of equation (10) is that the population at the beginning of year n (noted as *P[n]*) is equal to the population at the beginning of year n-1 (noted as *P[n-1]*) plus the transfers in (including the recruits) occurring in the whole of year n-1 (noted as *T[n]*) plus the total attrition in year n-1 (noted as *A[n]*).

one might think of rearranging (10) as shown in (11) to forecast annual attrition volume.

$$A[n] = P[n-1] - P[n] + T[n] \qquad (11)$$

Formula (11) would then be generalized to a YOS-based attrition formula as follows:

$$A_m[n] = P_{m-1}[n-1] - P_m[n] + T_m[n] \qquad (8)$$

However, since $P_{m-1}[n-1]$, $P_m[n]$ and $T_m[n]$ are the population and transfers with m-1 or m YOS, where YOS is measured at the beginning of the year n-1 or n, correspondingly, $A_m[n]$ is the attrition volume with m YOS measured at the beginning of the year, not at the time of release.

2. Using the following formula from reference [1], one might think of forecasting total annual attrition volume as follows:

$$A[n] = \alpha[n](P[n-1] + 0.5T[n]) \qquad (12)$$

Note: The effective attrition rate applied to the transfers (including recruits) is shown to be approximately half that used for the existing population because the transfers are, on average, only present for half of the snapshot interval as discussed in [1].

Then how about attrition volume at each YOS level? It would be natural to think that formula (13), which is similar to formula (12), would be used for forecasting attrition volumes at m YOS, where YOS is measured at the release dates.

$$A_m[n] = \alpha_m[n]\left(P'_{m-1}[n-1] + 0.5 T'_m[n]\right) \tag{13}$$

However, if $A_m[n]$ is the forecast attrition volume for personnel with m YOS, where YOS is measured at release dates, then, $P'_{m-1}[n-1]$ would need to be the population with m-1 YOS measured at the release dates and $T'_m[n]$ would need to be the transfers with m YOS measured at the release dates. Again, these numbers do not exist, thus this way is also not feasible.

3.1.2.2   The Way in New Procedures

3.   The new procedures use formula (14) for calculating the historical yearly attrition rate from a single year of data, as derived in [1].

$$\alpha_m = \frac{a'_m[n]}{\frac{1}{2}P_{m-1}[n-1] + \frac{1}{2}P_m[n-1] + \frac{1}{3}T_m[n] + \frac{1}{6}T_{m+1}[n]} \tag{14}$$

For multiple years of historical data, the equation is manipulated such that the attrition rate α_m is replaced by the calculated $WAAR(\alpha_m)$ from Step 2 in Table 1. Then $A_m[n]$ (the predicted attrition volume in year n-1 with m YOS measured at release dates) is solved by formula (6).

$$A_m[n] = WAAR(\alpha_m) \times \left(\frac{1}{2}P_{m-1}[n-1] + \frac{1}{2}P_m[n-1] + \frac{1}{3}T_m[n] + \frac{1}{6}T_{m+1}[n]\right) \tag{6}$$

The key change in the new procedures is the use of both $a'_m[n]$ and $a_m[n]$ for forecasting; the previous procedures used only $a_m[n]$. In the new procedures:

a. $a_m[n]$ is used for calculating $WAAR(\alpha_{m-1,m})$ as in formula (4) and then $P_m[n]$ is forecasted using formula (3); and

b. $a'_m[n]$ is used to calculate $WAAR(\alpha_m)$ as in formula (1), which is used to forecast attrition volume $A_m[n]$ using formulae (6) and (7).

This method of forecasting attrition at a given YOS where YOS is measured at the release dates was missing from previous analyses. The new procedures enable us to more accurately predict attrition volume at specific YOS points.

3.1.3   Example

The following example is used to explain why the new procedures should be used instead of the previous procedures.

Suppose it is the beginning of FY 07/08, and the number of the releases for the rest of the year must be predicted. Of particular interest is the predicted volume of RegF members that will be released during FY 07/08 with 20 YOS[4], which is denoted as $A_{20}[2008]$.

In order to forecast $A_{20}[2008]$, historical data is needed to calculate the historical attrition rate at 20 YOS. DGMPRA maintains a database containing CF personnel data dating back to FY 82/83. The choice of historical period(s) to use is a question analysts have faced on various attrition analyses. A three-step approach on the choice of historical data to use for attrition analyses has been presented in Fang and Bender (2008) [8]. For this example, historical data from FY 06/07 was chosen for forecasting RegF attrition in FY 07/08.

According to Table 1 and Table 2, $a_{20}[2007]$ (the number of releases in year 2006 with 20 YOS, where YOS is measured at the beginning of year 2007) is needed for the previous procedure, while the new procedure uses not only $a_{20}[2007]$ but also $a'_{20}[2007]$.

The value $a_m[n]$ (the total number of members who released in year n-1 with m YOS measured on the beginning of year n) is actually composed of two parts. For example $a_{20}[2007]$ can be written as follows,

$$a_{20}[2007] = A + B \tag{15}$$

where A is the number of members who released in FY 06/07 with 20 YOS measured at the beginning of FY 07/08 (Apr 1st of FY 07/08) and whose real YOS on their release date was 20; and B is the number of members who released in FY 06/07 with 20 YOS measured at the beginning of FY 07/08, but whose real YOS on their release date was 19.

Therefore, $a_{20}[2007]$ actually includes members with 19 and 20 YOS at the time of release. This is not exactly what is required. Similarly $a_{21}[2007]$ consists of two components:

$$a_{21}[2007] = C + D \tag{16}$$

where C is the number of members who released in FY 06/07 with 21 YOS measured at the beginning of FY 07/08 and whose real YOS on their release date was 21; and D is the number of people who released in FY 06/07 with 21 YOS measured at the beginning of FY 07/08, but whose real YOS was 20 on their release date.

---

[4]   At 20 YOS (with the previous Terms of Service), CF members have completed their Intermediate Engagement (IE). At this point, the CF experiences a high attrition rate because many of its members choose to retire rather than serve on an Indefinite Period of Service (IPS).

The new procedures use $a'_m[n]$ (number of releases in year n-1 with m YOS measured at the time of release). For our example, the real number of members who released in FY 06/07 with 20 YOS on their release date is:

$$a'_{20}[2007] = A + D \tag{17}$$

where A is from equation (16) and D is from equation (17).

Comparing equations (15) and (17), it is quite possible that B≠D. At the 20 YOS point, B is most likely less than D (i.e., B<D). Due to the exit gate at the 20 YOS point, more people would be likely to release at 20 YOS than 19 YOS (assuming that there is not much difference in the populations with 20 YOS and 19 YOS). Therefore, $a_{20}[2007]$ is quite possibly less than $a'_{20}[2007]$,

$$a_{20}[2007] < a'_{20}[2007] \tag{18}$$

Analysis of historical data proved this was true and that the same relationship existed across all years since 1982, that is $a_{20}[1982] < a'_{20}[1982]$, $a_{20}[1983] < a'_{20}[1983]$, …. $a_{20}[2006] < a'_{20}[2006]$.

The previous procedures used the historical attrition rates calculated from $a_m[n]$ to forecast attrition, while the new forecasting procedures use the historical attrition rates calculated from $a'_m[n]$. Although there are several other steps involved in performing the forecast, it is likely that the predictions of attrition volume at the 20 YOS point will differ between the two procedures. For the 20 YOS point, it is most likely that the attrition volume predicted by the previous procedures will underestimate the true attrition due to the fact that typically $a_{20}[n] < a'_{20}[n]$. In general, the difference between the previous procedures and the new procedures will exist for the other YOS points as well. As a result, the new procedures provide a more precise way to forecast the true attrition by YOS.

3.2   Forecasting Total Annual Attrition

Currently, there are two ways to forecast total annual attrition:

$$A[n] = \alpha[n](P[n-1] + \frac{1}{2}T[n]) \tag{12}$$

and

$$A[n] = \sum_{m=0}^{\max(YOS)} A_m[n] \tag{19}$$

where $A_m[n]$ is the predicted attrition volume at m YOS.

The difference between, and the applications of, these two methods have not been previously documented. Both methods are appropriate in some instances. When predicting yearly attrition using equation (12), the information required is the attrition rate α, the population at the beginning of the year ($P[n-1]$) and the planned recruitment for the year ($T[n]$). The YOS profile of the population is not taken into account. When predicting yearly attrition using equation (19), first the predicted attrition volume at each YOS ($A_m[n]$) needs to be calculated and then summed. The data needed to forecast $A_m[n]$ is not only the general information about the population and recruitment as in equation (12), but also more detailed data on the population profile by YOS, the transfers (including recruitments) by YOS and the attrition rate by YOS. Statistically speaking, more detailed information (assuming this information is accurate) yields better forecasts. Thus it is expected that, in general, forecasts based on formula (19) will produce more accurate predictions of yearly attrition. Comparisons of the two methods on total annual attrition predictions can be found in Annex B.3.

It is recommended that formula (19) be used to forecast annual attrition volume if it is possible. However, care must be taken if the population of the segments is small, because formula (19) breaks the population down into the individual YOS sub-groups and calculates the attrition rates at each YOS. The uncertainty of these calculated attrition rates will be high because the population of the sub-groups at each YOS will be even smaller. Therefore, when the size of the segment of interest is small or when the YOS profile of that segment is not available, formula (12) can be used for forecasting annual attrition. Furthermore, methodological research in Fang 2008 [10] provided a solution on modelling attrition for the small segments.

4   Validation/Comparisons

4.1   A Validation of New Forecasting Procedures

This section presents a real world application. Suppose that at the beginning of FY 07/08, the new procedures had been used for forecasting attrition for FY 07/08 and, as was done in Section 3.1, historical data from FY 06/07 was used to forecast RegF attrition in FY 07/08. After the end of the year, the forecasted attrition volumes at each YOS could then be compared with actual attrition volumes for FY 07/08. Figure 2 provides a visual comparison between the predicted attrition volume using the new forecasting procedures and actual attrition volume for FY 07/08. (The values are listed in Table 4 in Section 4.2.1).
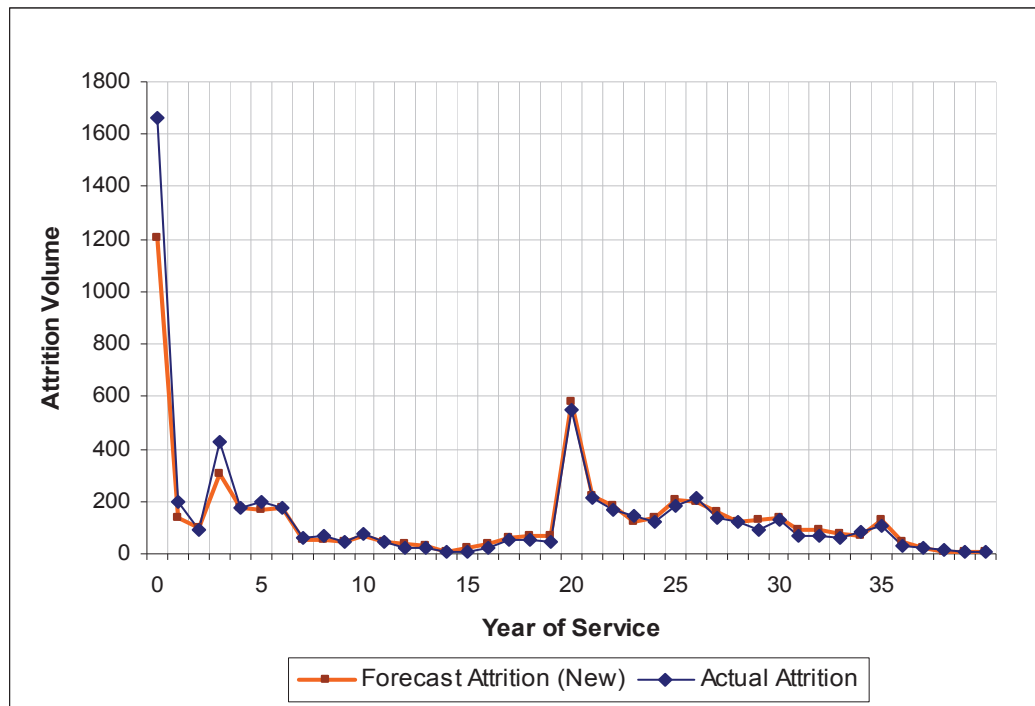
Figure 2: Comparison between Forecasted Attrition using the
New Procedures and Actual Attrition for FY 07/08

Figure 2 shows a strong agreement between the forecast attrition volume using the new procedures and the actual attrition volume, except for the 0 YOS point. At the 20 YOS point (a YOS of special interest), the predicted attrition was 580 and the actual attrition was 551 (see Table 4). For the 0 YOS point (another YOS of special interest), although the new procedures have improved the attrition prediction as compared to the previous predictions (as shown in Section 4.2.1), there is still a difference between forecast and actual attrition. The briefing notes on the *Cohort Analysis of First Year Attrition* by Fang and Latchman (2008) [11-12] explored this area.

For a formal comparison, a statistical test needs to be conducted. According to the data structure and characteristics, a paired-t-test was chosen for comparing the difference between the predicted attrition volume and the actual attrition volume at each YOS point. The t statistic is -0.86 with 40 degrees of freedom. The P-value associated with this test is 0.39, thus the test failed to find a statistically significant difference between the forecast attrition volumes and the actual attrition volumes.

Table 3: Paired T-test between Forecasts and Actual Attrition by YOS

	T Statistic	Degrees of Freedom	P-value
Paired t-test	-0.86	40	0.39

The statistical test showed the attrition model produced using the new procedure is a good fit to the actual RegF attrition behaviour experienced during that year. This provides a validation of the new forecasting procedures.

As it was mentioned earlier, the forecasting procedures can be applied to attrition forecasting for the future calendar year (CY) or FY. A similar analysis for the CY 2006 is provided in Annex B to illustrate forecasting attrition for a CY and to provide an additional example.

4.2   Comparisons between the Two Procedures

The old and new procedures will be compared in this section. It should be mentioned that the old procedures provided a way to forecast the net attrition by YOS (where YOS is measured at the beginning of the year). The new procedures produce forecasts of the true attrition by YOS (where YOS is measured at release date), which was missing from the old methodology. The old procedures are still valid for forecasting the net attrition by YOS. However, as discussed earlier, attrition by YOS, where YOS is measured at release date, is a more useful statistic because it better reflects the CF members' attrition behaviours, and it should be used when forecasting and reporting CF attrition.

The Personnel Generation Research Section maintains databases of RegF personnel dating back to 1982. The data can be split into several periods that differ in terms of attrition behaviour. Fiscal year rather than calendar year is frequently requested for attrition analyses. Various historical periods in the CF historical databases that are available for forecasting attrition *for FY 07/08* are listed below. Subject matter expertise has been consulted on the separation of the historical periods.

   a.   FY 82/83 to FY 06/07: the whole period (for which RegF personnel data are available for forecasting FY 07/08);

   b.   FY 82/83 to FY 91/92: pre-Force Reduction Program[5] (FRP);

---

[5]   In the early to mid-1990s, the Department of National Defence encouraged members to take early retirement. This was part of a program to reduce the complement of the Canadian Forces. Members were provided by DND with details of the early retirement incentives. The attrition behaviours were unusual.

c.   FY 92/93-FY 96/97: FRP years;

    d.   FY 97/98-FY 06/07: post FRP;

    e.   FY 01/02-FY 06/07: CF expansion years; and

    f.   FY 06/07: the most recent year, which is the period of the choice for forecasting
         attrition for FY 07/08[6].

For the purposes of objective comparisons, the predictions from the previous and new forecasting
procedures were compared using not only the data from the selected period, i.e., FY 06/07, but
also the data from other historical periods. The questions of interest are the following:

    a.   Do the new procedures produce more accurate forecasts than the previous procedures
         using data from the period of choice (FY 06/07)?

    b.   Do the new procedures produce more accurate forecasts than the previous procedures
         using data from other historical periods?

The following sections will address these questions. Annex B and C provide additional
comparisons between the new and old procedures.

---

[6]   The reason the period of FY 06/07 is recommended for forecasting FY 07/08 attrition was discussed
      in section 3.1.

4.2.1   Forecasts Based on the Historical Period of Choice: FY 06/07

Forecasts for FY 07/08 using both the new and the previous procedures were performed first based on historical data from the period of the choice – FY 06/07. These forecasts are compared against the actual releases for FY 07/08 in Table 4.

Table 4: Comparisons between New and Previous Forecasts and the Actual Releases for FY 07/08 (The forecasts are based on the historical period of choice: FY 06/07)

YOS	New Procedures	Previous Procedures	Actual Releases	YOS	New Procedures	Previous Procedures	Actual Releases
0	1205	759	1666	21	222	393	210
1	135	521	201	22	179	150	167
2	96	113	94	23	123	193	148
3	305	312	425	24	140	83	120
4	175	150	173	25	206	174	186
5	169	185	201	26	200	230	214
6	175	157	172	27	158	185	135
7	51	102	58	28	120	128	120
8	50	45	66	29	129	106	92
9	46	55	47	30	134	152	131
10	70	56	79	31	88	96	68
11	43	57	48	32	93	94	71
12	37	33	25	33	77	83	60
13	28	33	24	34	67	73	82
14	9	28	10	35	131	124	104
15	25	11	9	36	43	71	33
16	38	29	23	37	19	29	20
17	59	49	51	38	8	13	13
18	65	66	52	39	7	8	4
19	70	68	47	40	5	2	5
20	580	338	551	Total	5580	5554	6005

4.2.1.1   Comparisons of Predicted Attritions at 0 and 20 YOS

For comparison, error rates were calculated. The error rate was defined as following:

$$ErrorRate = \frac{Forecast - Actual}{Actual} \tag{20}$$

The comparisons for the 0 YOS and 20 YOS points are shown in Table 5.

Table 5: Comparisons of Predicted Attritions at 0 and 20 YOS for FY 07/08
(The forecasts are based on the choice of historical period: FY06/07)

YOS	New Predictions (Error Rate Percent)	Previous Predictions (Error Rate Percent)	Actual Attrition
0 YOS	**1205 (-27.7)**	759 (54.4)	1666
20 YOS	**580 (5.2)**	338 (-38.6)	551

* More accurate numbers are bolded

The predicted FY 07/08 attrition volumes for the 0 YOS point are 1205 and 759 from the new and previous forecasting procedures respectively, while the actual attrition volume was 1666. The error rate of the new prediction at the 0 YOS point is less than that of the previous prediction. However, there is still a non-negligible difference between the new prediction and the actual attrition. As mentioned earlier, early attrition has been investigated in [11-12]; one of the findings from this research was that the RegF has experienced increasing first year attrition rates (i.e., attrition rates for members with 0 YOS) since FY 01/02. This increasing first year attrition rate may caused by the changing quality of new recruits, which was impacted by the lower selection standard, e.g., the waiver of physical fitness test in the selection; and the accelerated enlistment, e.g., the goal is 30% of recruits should be enrolled within a week.

The predicted FY 07/08 attrition volumes at the 20 YOS point from the new and previous procedures is 580 and 338 respectively; the actual attrition volume at the 20 YOS point in FY 07/08 was 551. As with the 0 YOS point, the predicted attrition volume produced by the new procedures is closer to the actual attrition volumes at the 20 YOS point than that produced by the previous procedures. As expected (and discussed in section 3.1), the attrition rate at 20 YOS as calculated by the previous procedures (where YOS is measured at the end of year) is lower than that calculated with the new procedures (where YOS is measured at release date).

In summary, the new procedures produce predicted attrition volumes (for FY 07/08) that are closer to the actual attrition at both 0 and 20 YOS when the forecasts are based on the data from the historical period of the choice- FY 06/07.

4.2.1.2   Comparisons of Predicted Attrition across all YOS Points

It is of interest to know whether the overall performance of the new forecasting procedures is better than that of the previous procedures. In order to compare the forecasts across all YOS points, the Root Mean Squared Errors (RMSE) were calculated for the two forecasting procedures. The RMSE is defined as the following:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (Forecast_i - Actual_i)^2}{n}} \tag{21}$$

where i represents each YOS point.

The smaller the RMSE, the better the prediction. The RMSE for the FY 07/08 predictions produced by new procedures is 77, which means on average there is a difference of 77 releases between the prediction and the actual releases at YOS level. The RMSE for the previous procedures is 159. This shows that the predictions produced by the new procedures are more accurate across different YOS points than those produced by the previous procedures, if historical data of FY 06/07 are used.

4.2.1.3   Summary

Overall, the new forecasting procedures produce better predictions than the previous procedures when using data from FY 06/07 as the data source for the models. The spikes in attrition at the 0 and 20 YOS points are also more accurately modeled by the new procedures.

4.2.2   Forecasts Based on Historical Data from Other Periods

Attrition forecasts based on data from the following three other historical periods were also calculated and compared in Table 6 with actual releases for FY 07/08: the whole historical period available (FY 82/83 to FY 06/07); the post FRP period (FY 97/98 to FY 06/07); and the CF expansion period (FY 01/02 to FY 06/07). The other two periods, pre-FRP and FRP have rarely been used for recent CF attrition forecasts because they are less relevant to the current CF situation. Nonetheless the comparisons between the new and old predictions based on data from these two periods were also performed and results can be found in Annex C.

Table 6: Comparisons between New and Previous Forecasts and Actual Releases for FY 07/08

YOS	Actual Releases	Whole Historical Period		CF Expansion Period		Post FRP Period	
		New Procedures	Previous Procedures	New Procedures	Previous Procedures	New Procedures	Previous Procedures
0	1666	1164	832	956	680	1005	723
1	201	310	526	186	383	188	394
2	94	202	240	119	142	131	143
3	425	465	456	355	335	387	371
4	173	177	178	139	128	138	132
5	201	181	200	127	149	131	148
6	172	205	203	171	129	190	138
7	58	94	135	55	118	66	134
8	66	75	78	48	42	65	55
9	47	71	72	48	54	63	65
10	79	88	80	74	64	87	80
11	48	55	80	41	62	50	75
12	25	33	35	30	28	33	32
13	24	27	31	23	28	25	31
14	10	17	23	14	20	15	21
15	9	21	13	16	10	17	10
16	23	40	27	30	21	32	22
17	51	59	55	46	40	50	44
18	52	67	68	50	51	50	54
19	47	85	80	51	51	49	49
20	551	381	244	471	283	482	288
21	210	174	274	200	319	205	330
22	167	148	122	157	134	161	135
23	148	114	173	125	192	121	191
24	120	110	75	102	72	105	73
25	186	169	139	153	131	151	129
26	214	148	177	139	164	137	161
27	135	129	132	118	128	116	124
28	120	107	123	93	106	94	108
29	92	95	94	80	77	80	79
30	131	117	115	91	95	91	95
31	68	83	90	64	70	63	69
32	71	77	80	58	60	60	60
33	60	64	68	53	56	51	56
34	82	65	63	62	54	61	53
35	104	81	89	115	112	112	107
36	33	33	46	40	58	40	58
37	20	17	22	22	29	22	30
38	13	7	11	10	17	11	18
39	4	1	4	7	11	7	12
40	5	0	0	4	3	4	4
Total	6005	5556	5553	4743	4706	4946	4901

4.2.2.1 Comparison of Predicted Attrition at 0 and 20 YOS

Whatever the historical period chosen, Tables 7 and 8 show that for both the 0 YOS and the 20 YOS points, the FY 07/08 predictions produced by the new procedures are consistently closer to the actual attrition than those produced by the previous procedures.

Table 7: Forecast FY 07/08 Attrition at 0 YOS using the New and Previous Procedures

Historical Data Used	New Predictions (Error Rate Percent)	Previous Predictions (Error Rate Percent)	Actual Attrition
Whole period	**1164 (-30.1)**	832(-50.1)	
Pre-FRP	**1253 (-24.8)**	891(-46.5)	
FRP	**1330 (-20.2)**	971 (-41.7)	1666
Post FRP	**1005 (-39.7)**	723 (-56.6)	
CF expansion	**956 (-42.6)**	680 (-59.2)	

*More accurate numbers are bolded.

Table 8: Forecast FY 07/08 Attrition at 20 YOS using the New and Previous Procedures

Historical Data Used	New Predictions (Error Rate Percent)	Previous Predictions (Error Rate Percent)	Actual Attrition
Whole period	**381 (-30.8)**	244(-55.7)	
Pre-FRP	**188(-65.8)**	119(-78.5)	
FRP	**478(-13.2)**	346(-37.1)	551
Post FRP	**482 (-12.5)**	288 (-47.7)	
CF expansion	**471(-14.6)**	283 (-48.6)	

*More accurate numbers are bolded.

4.2.2.2 Comparison of Predicted Attrition Across All YOS Points

Table 9 shows that the RMSE of the new procedures are consistently less than those of the previous procedures, regardless of the historical period chosen. This demonstrates that, from an overall performance perspective, the new procedures produce better predictions than the previous procedures across different YOS points.

Table 9: Comparison of the Root Mean Square Errors of the New and Previous Procedures

Historical Data Used	RMSE	
	New Procedures	Previous Procedures
Whole period	**97**	157
Pre-FRP	**84**	148
FRP	**106**	159
Post FRP	**114**	166
CF expansion	**77**	159

*More accurate numbers are bolded.

4.2.2.3   Comparison of Predicted Total Attrition Volume

Finally, Table 10 shows that for the total FY 07/08 attrition prediction, the error rates for the new procedures are slightly smaller than those from the previous forecasting procedures for all the other historical periods except the pre-FRP period.

Table 10: Forecast Total Attrition Volume for FY 07/08 using New and Previous Procedures

Historical Data Used	New Prediction (Error Rate Percent)	Previous Prediction (Error Rate Percent)	Actual Attrition
Most recent	**5580 (-7.1)**	5554 (-7.5)	
Whole period	**5556 (-7.5)**	5553 (-7.5)	
Pre-FRP	5223 (-12.9)	**5245 (-12.7)**	
FRP	**7006 (16.7)**	7050 (17.4)	**6005**
Post FRP	**4946 (-17.6)**	4901 (-18.4)	
CF expansion	**4743 (-21.0)**	4706 (-21.6)	

*More accurate numbers are bolded.

Thus, there appears to be not much difference between the two procedures when forecasting total annual attrition. But at individual YOS points, the new procedures provide predictions which better reflect the RegF attrition behaviour. Therefore, the new forecasting procedures are recommended for both forecasting attrition at each YOS and forecasting total annual RegF attrition.

4.2.2.4   Summary

In general, forecasts produced by the new procedures better reflect the CF attrition behaviour than those produced by the previous procedures. The findings from the comparisons provided consistent empirical evidence of the improved performance of the new procedures. The theoretical reasons why the new procedures are appropriate and should be used for forecasting RegF attrition have been presented in Section 3.

However, it is not expected that the new procedures will always produce precise forecasts because many factors may "influence" or "impact" the accuracy of predictions. For example, there can be policy changes related to attrition (for example, changes to the Terms of Service), external economic incentives, and changes in attrition behaviour related to operations/personnel tempos.

5   Conclusion

In this report, new procedures are proposed for forecasting attrition that enable analysts to forecast the true attrition by YOS (where YOS is measured at the time of release). The proposed procedures were described in detail, then validated and compared using actual RegF data.

The forecasts based on the new procedures show a strong agreement with the actual attrition volume. They also reflect the attrition at both 0 and 20 YOS points better than the previous procedures, and the RMSE demonstrates that, from an overall performance perspective, the new procedures produce better predictions than previous procedures across different YOS points.

Based on these findings, it is recommended that the new procedures be adopted for future CF attrition forecasts. The methodology can also be applied to different segments within the CF, such as NCMs and Officers; Army, Navy and Air Force; and different military branches and occupations. However, particular attention needs to be drawn when the population of the segments is small, because the large variability is always an issue analysts face when dealing with the small segments. Ways of addressing small segments in attrition analyses are discussed in references [8] and [10].This new procedure has been used for a variety of CF attrition analyses, such as attrition analyses for AMORs and for the Annual Report on Regular Force Attrition [9].

This study contributes to DGMPRA's continuing efforts to develop rigorous methodologies in attrition analysis. This document is intended to keep analysts who address personnel issues updated on the latest methodology development on forecasting CF attrition, and to improve the transparency and consistency of attrition analyses practiced by different research groups. Given that attrition forecasting is vitally important for a number of relevant human resources initiatives, such as recruitment, promotion, planning and budget management, this work will have a positive impact on all these areas of CF human resources.

References

[1]   Okazawa, S. (2007). *Measuring Attrition Rates and Forecasting Attrition Volume*. CORA TM 2007-02. Defence R&D Canada – CORA.

[2]   Ewashko, T.A., Lee, A.D., Platz, A.C. (1991). *An Examination of the Structure and Behaviour of Attrition within the Canadian Forces in Engineering Officer Occupations*. D MAN A Staff Note 1991/02.

[3]   Provencher, M., Wentzell, T.E., Collin, I.A., Kerzner, L.F. (2001). *Attrition Forecasts: A First Attempt*. DOR (Corp) Research Note RN 2001/02.

[4]   Provencher, M. (2001). *PARRA Attrition Forecasts: A More Refined Model*. ORD Project Report PR 2002/03.

[5]   Moorhead, P. and Zegers, A. (2003). C*omparing Observed and Stable Population Profiles within the Canadian Forces*. DOR(Corp) Research Note RN 2003/11.

[6]   Moorhead, P. (2004). *Determining CF Stable Population Profiles Via Survival Analysis Techniques*. DOR(Corp) Research Note RN 2004/02.

[7]   Moorhead, P. (2006). *A Methodology for Determining Stable Age and Years of Service Profiles for CF Occupations*. DRDC CORA TM 2006-23. Defence R&D Canada – CORA.

[8]   Fang, M., Bender, P. (2008). *CF Attrition Forecasts: What One Should Know*. DRDC CORA TM 2008-060. Defence R&D Canada – CORA.

[9]   Straver M.C., Fang M, Koundakjian K, Latchman S.A., Michaud K. (2009) *Annual Report on Regular Force Attrition 2007/2008*.

[10]   Fang, M. (2008). *Systematic Review on Attrition*. TTCP TP3 Workforce Modelling Meeting, May 2008, Ottawa, Ontario.

[11]   Fang, M., Latchman, S. (2008). *Cohort Analysis of New Recruit Attrition*. Briefing Note for CMP, 9 April 2008.

[12]   Fang, M., Latchman, S. (2008). *Updated Cohort Analysis of New Recruit Attrition*. Briefing Note for CMP, 25 September 2008.

This page intentionally left blank.

Annex A New Procedures for Forecasting Using the Least Squares Approach

As stated in Section 2.2, in order to get an overall estimate for the attrition rate from multiple years of historical data, the Weighted Average or Least Squares approach can be used. In the main body of this report, the Weighted Average approach is applied in both new and old procedures. Table 11 presents an alternative, the new procedures for forecasting attrition using the Least Squares approach.

Table 11: New Procedures for Forecasting Attrition by YOS using the Least Squares Approach

Steps	Procedures
1	Using historical data, obtain: • $a'_m[n]$ - number of members who released in year n-1 with m YOS, where YOS is measured at release date, • $a_m[n]$ - number of members released in year n-1 with m YOS measured on Jan 1/Apr 1 of year n, • $p_m[n]$ - population with m YOS measured on Jan 1st/Apr 1st of year n, and • $T_m[n]$ - Transfers or/and recruits in year n-1 with m YOS measured on Jan 1st/Apr 1st of year n.
2	Calculate the YOS based least squares attrition rates LSAR (α_m) based on $a'_m[n]$ by using the following formulae: When m >0, $$LSAR(\alpha_m) = \frac{\sum_{n=1}^{N} a'_m[n] \times \left(\frac{1}{2} P_{m-1}[n-1] + \frac{1}{2} P_m[n-1] + \frac{1}{3} T_m[n] + \frac{1}{6} T_{m+1}[n] \right)}{\sum_{n=1}^{N} \left(\frac{1}{2} p_{m-1}[n-1] + \frac{1}{2} p_m[n-1] + \frac{1}{3} T_m[n] + \frac{1}{6} T_{m+1}[n] \right)^2}$$ When m=0, $$LSAR(\alpha_0) = \frac{\sum_{n=1}^{N} a'_0[n] \times \left(\frac{1}{2} P_0[n-1] + \frac{1}{2} T_0[n] + \frac{1}{6} T_1[n] \right)}{\sum_{n=1}^{N} \left(\frac{1}{2} p_0[n-1] + \frac{1}{2} T_0[n] + \frac{1}{6} T_1[n] \right)^2}$$

3	Calculate $P_{m-1}[n]$ and $P_m[n]$ using the following:

$$p_m[n] = \left(1 - LSAR(\alpha_{m-1,m})\right)p_{m-1}[n-1] + \left(1 - \frac{1}{2}LSAR(\alpha_{m-1,m})\right)T_m[n]$$

Where when m>0,

$$LSAR(\alpha_{m-1,m}) = \frac{\sum_{n=1}^{N} a_m[n] \times \left(P_{m-1}[n-1] + \frac{1}{2}T_m[n]\right)}{\sum_{n=1}^{N}\left(p_{m-1}[n-1] + \frac{1}{2}T_m[n]\right)^2}$$

When m=0,

$$LSAR(\alpha_0) = \frac{\sum_{n=1}^{N} a_0[n] \times \left(\frac{1}{2}T_0[n]\right)}{\sum_{n=1}^{N}\left(\frac{1}{2}T_0[n]\right)^2}$$ |
| 4 | Forecast volume of attrition for personnel with with m YOS, $A_m[n]$.

For the CF as a whole:

$$A_m[n] = LSAR(\alpha_m) \times \left(\frac{1}{2}P_{m-1}[n-1] + \frac{1}{2}P_m[n-1]\right)$$

For each occupation or the sub-population:

$$A_m[n] = LSAR(\alpha_m) \times \left(\frac{1}{2}P_{m-1}[n-1] + \frac{1}{2}P_m[n-1] + \frac{1}{3}T_m[n] + \frac{1}{6}T_{m+1}[n]\right)$$

It is notable that $LSAR(\alpha_m)$ is calculated from $a'_m[n]$. |

Annex B   Forecasting Attrition for Calendar Year 2006

In the main body of this report, empirical evidence of the performance of the new forecasting procedures was presented by using a fiscal year example. In this Annex, the new procedures are applied to a calendar year (CY) example, specifically CY 2006. This provides additional evidence regarding the performance of the new procedures.

Section B1 compares a CY 2006 forecast and actual attrition volume at each YOS. Section B2 compares actual attrition for the year to the forecast attrition using both the new and old procedures, and based on a variety of historical periods. Section B3 compares forecasts of total annual attrition volume using the two equations from Section 3.2, in particular equations (12) and (19)

B.1   Validation by Comparing Predictions and Actual Attrition for Year 2006
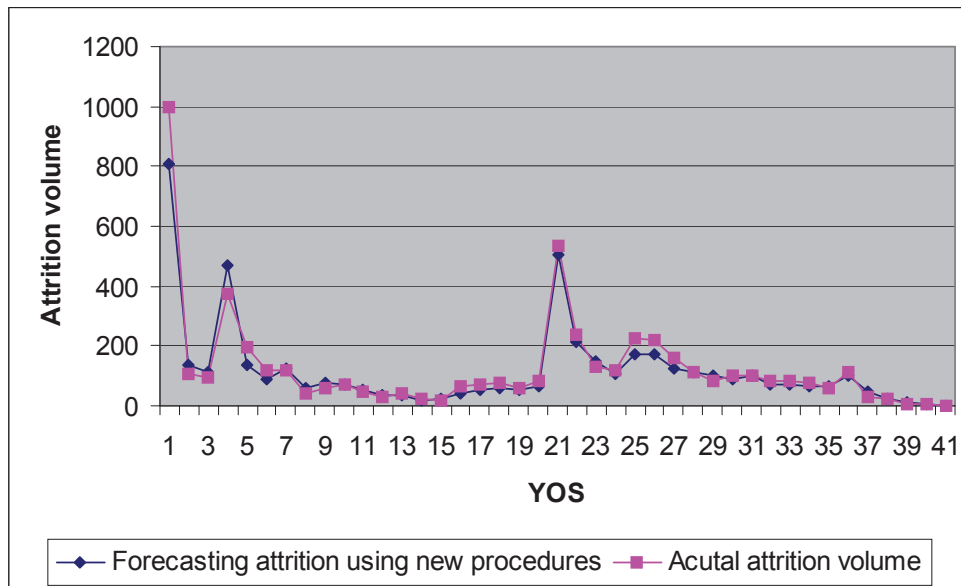
Figure 3: Comparison between Forecast Attrition Volume for CY 2006 using the New Procedures (based on historical data from CY 2005) and Actual Attrition Volume

The paired t-test shows there is no statistical significant difference between forecasts and actual attrition for 2006 (Table 12).

Table 12: Paired T-test between Forecasts and Actual Attrition by YOS

	T Statistic	**Degrees of Freedom**	**P-value**
Paired t-test	1.50	40	0.14

B.2 Comparisons of Attrition Predictions for CY 2006 Produced by the New and Previous Procedures

Table 13: Forecast CY 2006 Attrition at 20 YOS using New and Previous Procedures*

Historical Data Used	New Procedures (Error Rate Percent)	Previous Procedures (Error Rate Percent)	Actual Attrition in 2006
1983 – 2005	**388 (27.5)**	347(35.1)	
1983 – 1991	**192(64.1)**	173(67.7)	
1992 – 1996	**490 (8.4)**	468(12.5)	535
1997 – 2005	**505 (5.6)**	427(20.2)	
2001 – 2005	**481(10.0)**	414(22.6)	

*more accurate numbers are bolded

Table 14: Root Mean Square Errors for CY 2006 Attrition Forecasts by YOS using the New and Previous Procedures*

Historical Data Used	RMSE	
	Using New Procedures	Using Previous Procedures
1983 – 2005	**46**	71
1983 – 1991	**75**	91
1992 – 1996	**67**	94
1997 – 2005	**39**	66
2001 – 2005	**51**	73

*more accurate numbers are bolded

Table 15: Forecast Attrition by YOS for 2006 (based on historical data from CY 1983 to 2005)

YOS	New Procedures	Previous Procedures	Actual Attrition	YOS	New Procedures	Previous Procedures	Actual Attrition
0	945	763	999	21	176	193	239
1	219	375	105	22	135	187	130
2	163	158	93	23	97	87	116
3	544	449	376	24	182	135	223
4	192	244	196	25	199	218	218
5	128	171	119	26	142	155	161
6	127	123	118	27	129	137	111
7	77	90	42	28	117	114	85
8	85	72	60	29	111	120	99
9	76	94	72	30	127	119	102
10	56	73	46	31	91	101	81
11	36	32	27	32	91	89	81
12	32	40	42	33	78	87	76
13	17	23	22	34	63	71	57
14	26	13	16	35	68	67	113
15	51	39	65	36	33	46	29
16	66	62	70	37	14	20	21
17	70	74	80	38	4	8	8
18	67	69	59	39	1	2	4
19	102	82	82	40	0	0	2
20	388	347	535	Total	5325	5349	5180

Table 16: Forecast Attrition by YOS for 2006
(based on the historical data from CY 1983 to 1991)

YOS	New Procedures	Previous Procedures	Actual Attrition	YOS	New Procedures	Previous Procedures	Actual Attrition
0	996	806	999	21	121	115	239
1	243	404	105	22	106	140	130
2	177	175	93	23	77	68	116
3	587	477	376	24	181	128	223
4	210	276	196	25	206	221	218
5	139	186	119	26	135	151	161
6	120	123	118	27	120	127	111
7	82	87	42	28	112	109	85
8	87	79	60	29	101	111	99
9	72	91	72	30	120	114	102
10	52	66	46	31	85	95	81
11	33	29	27	32	81	80	81
12	26	34	42	33	63	73	76
13	13	18	22	34	46	53	57
14	21	10	16	35	40	43	113
15	34	29	65	36	22	29	29
16	48	42	70	37	8	12	21
17	38	49	80	38	2	4	8
18	36	36	59	39	0	1	4
19	34	31	82	40	0	0	2
20	192	173	535	Total	4866	4895	5180

Table 17: Forecast Attrition by YOS for CY 2006
(based on historical data from CY 1992 to 1996)

YOS	New Procedures	Previous Procedures	Actual Attrition	YOS	New Procedures	Previous Procedures	Actual Attrition
0	1111	851	999	21	209	249	239
1	317	564	105	22	164	227	130
2	206	217	93	23	123	108	116
3	510	467	376	24	206	157	223
4	195	234	196	25	235	250	218
5	131	179	119	26	184	199	161
6	143	138	118	27	172	183	111
7	84	99	42	28	143	146	85
8	90	76	60	29	158	166	99
9	93	107	72	30	173	155	102
10	63	85	46	31	130	141	81
11	45	39	27	32	138	133	81
12	42	51	42	33	129	136	76
13	24	32	22	34	99	117	57
14	38	18	16	35	87	89	113
15	90	63	65	36	43	62	29
16	114	111	70	37	20	27	21
17	133	132	80	38	6	12	8
18	144	144	59	39	1	3	4
19	291	217	82	40	0	0	2
20	490	468	535	Total	6774	6852	5180

Table 18: Forecast Attrition by YOS for CY 2006
(based on historical data from CY 2001 to 2005)

YOS	New Procedures	Previous Procedures	Actual Attrition	YOS	New Procedures	Previous Procedures	Actual Attrition
0	736	624	999	21	200	220	239
1	134	248	105	22	143	206	130
2	102	95	93	23	109	94	116
3	425	338	376	24	157	125	223
4	132	158	196	25	168	186	218
5	80	109	119	26	123	132	161
6	107	88	118	27	112	122	111
7	46	69	42	28	95	94	85
8	54	38	60	29	81	89	99
9	54	74	72	30	87	88	102
10	48	62	46	31	64	70	81
11	27	27	27	32	60	59	81
12	27	33	42	33	59	68	76
13	15	19	22	34	63	59	57
14	21	12	16	35	107	94	113
15	35	28	65	36	49	66	29
16	47	43	70	37	26	35	21
17	50	50	80	38	11	18	8
18	45	48	59	39	7	13	4
19	52	47	82	40	2	2	2
20	481	414	535	Total	4441	4464	5180

B.3   Comparisons of Annual Attrition Predictions for CY 2006

Table 19: Comparison of Annual Attrition Predictions for CY 2006 using Methods Listed Below

Historical Data Used	New Predictions (Error rate Percent) based on $\sum_{m=0}^{max(YOS)} A_m[n]$	Predictions (Error Rate Percent) based on $A[n] = \alpha(p[n-1] + \frac{1}{2} r[n])$	Actual Attrition
1983 – 2005	5325 (2.8)	4759 (-8.2)	
1983 – 1991	4866 (-6.1)	4691 (-9.4)	
1992 – 1996	6774 (30.8)	5611 (8.3)	5180
1997 – 2005	4816 (-7.0)	4277 (-17.3)	
2001 – 2005	4441 (-14.3)	4180 (-19.3)	

* More accurate numbers are bolded.

Annex C   Attrition Forecasts Based on Data from Pre-FRP and FRP Period

In Section 4.2.2, empirical evidence of the performance of the new forecasting procedures was given by forecasting the attrition of FY 07/08. Data from different historical periods were used, but since pre-FRP and FRP periods are less relevant to current CF situation, the forecasts based on data from these two periods were not listed in the main body of the report. However, the comparisons between the new and old predictions based on the data from these two periods was also performed and the detailed data are presented here to provide additional empirical evidence.

Table 20: Comparisons between Forecasts from the New,
Old Procedures and the Actual Releases for FY 07/08
(Forecasts are based on data from Pre-FRP period: FY 82/83 to FY 91/92)

YOS	New Procedures	Old Procedures	Actual Releases	YOS	New Procedures	Old Procedures	Actual Releases
0	1253	891	1666	21	117	156	210
1	361	593	201	22	116	95	167
2	231	278	94	23	91	130	148
3	516	498	425	24	108	69	120
4	194	203	173	25	173	141	186
5	197	220	201	26	137	176	214
6	194	216	172	27	121	118	135
7	102	125	58	28	105	119	120
8	78	86	66	29	86	88	92
9	67	71	47	30	115	107	131
10	82	74	79	31	78	90	68
11	50	74	48	32	71	73	71
12	27	30	25	33	54	60	60
13	21	24	24	34	51	52	82
14	14	18	10	35	51	60	104
15	14	10	9	36	24	32	33
16	30	19	23	37	11	15	20
17	34	39	51	38	3	6	13
18	36	35	52	39	0	1	4
19	30	35	47	40	0	0	5
20	188	119	551	Total	5233	5245	6005

Table 21: Comparisons between Forecasts from the New and Old Procedures and the Actual Releases for FY 07/08 (Forecasts are based on data from the FRP period: FY 92/93 to FY 96/97)

YOS	New Procedures	Old Procedures	Actual Releases	YOS	New Procedures	Old Procedures	Actual Releases
0	1330	971	1666	21	210	364	210
1	451	668	201	22	186	144	167
2	247	325	94	23	147	219	148
3	434	468	425	24	128	91	120
4	183	172	173	25	197	162	186
5	192	209	201	26	191	213	214
6	236	230	172	27	170	177	135
7	104	156	58	28	134	157	120
8	81	84	66	29	136	129	92
9	88	83	47	30	159	158	131
10	100	93	79	31	118	120	68
11	69	96	48	32	115	120	71
12	44	45	25	33	103	105	60
13	37	41	24	34	101	98	82
14	26	33	10	35	98	120	104
15	38	22	9	36	42	59	33
16	69	48	23	37	24	30	20
17	116	100	51	38	12	17	13
18	150	147	52	39	2	8	4
19	257	221	47	40	0	1	5
20	478	346	551	Total	7006	7050	6005

List of symbols/abbreviations/acronyms/initialisms

$A[n]$	Predicted attrition volume for year n-1, for all YOS
$P[n]$	Population on Jan 1st/ Apr 1st of year n
α	Yearly attrition rate
$r[n]$	Total recruitment occurring for year n-1
$a'_m[n]$	Number of releases in year of n-1, where the released members have m YOS measured at the time of release
$a_m[n]$	Number of releases in year of n-1, where the released members have m YOS, where YOS is measured on Jan 1st /Apr 1st of year n
$A_m[n]$	Predicted attrition volume for year n-1, for members with m YOS measured on Jan 1st/ Apr 1st of year n
$P_m[n]$	Population with m YOS on Jan 1st/ Apr 1st of year n
$T_m[n]$	Transfers/Recruits in year n-1 with m YOS, where YOS is measured on Jan 1st/ Apr 1st of year n
AMOR	Annual Military Occupational Reviews
CF	Canadian Forces
CY	Calendar Year
DGMPRA	Director General Military Personnel Research and Analysis
DPGR	Director Personnel Generation Requirements
DRDC	Defence Research and Development Canada
FRP	Force Reduction Program
FY	Fiscal Year
IE	Intermediate Engagement
IPS	Indefinite Period of Service
LSAR	Least Squares Attrition Rate
NCM	Non-Commissioned Member
RegF	Regular Forces
RMSE	Root Mean Square Error
SIP	Strategic Intake Plan
TOS	Terms of Service
WAAR	Weighted Average Attrition Rate

$WAAR(\alpha_m)$ YOS based Weighted Average Attrition Rate

$WAAR(\alpha_{m-1,m})$ Net Weighted Average Attrition Rate

YOS   Years of Service

Distribution list

Document No.: DGMPRA TM 2009-025

LIST PART 1: Internal Distribution by Centre

1	CMP
1	Asst CMP
1	DG CORA
1	DRDC CORA – Chief Scientist
1	DRDC CORA Library
2	DRDKIM Library
7	DG CORA – Section Heads
1	DG CORA – Team Leaders
1	DGMPRA
1	DGMPRA – Chief Scientist
1	DGMPRA– Deputy DG
1	DGMPRA – Personnel Generation Research – Section Head
1	DGMPRA – Personnel and Family Support Research – Section Head
1	DGMPRA – Organizational and Operational Dynamics – Section Head
1	DGMPRA – Team Leaders
1	DGMP
1	DPGR
1	Library and Archives Canada
3	Author
2	Spares

30   TOTAL LIST PART 1

LIST PART 2: External Distribution by DRDKIM

1   Library and Archives Canada

1   TOTAL LIST PART 2

31   TOTAL COPIES REQUIRED

This page intentionally left blank.

DOCUMENT CONTROL DATA			
(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)			
1. ORIGINATOR (The name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Centre sponsoring a contractor's report, or tasking agency, are entered in section 8.) DGMPRA 101 Colonel By Drive Ottawa, Ontario K1A 0K2		2. SECURITY CLASSIFICATION (Overall security classification of the document including special warning terms if applicable.) UNCLASSIFIED (NON-CONTROLLED GOODS) DMC A REVIEW:GCEC JUNE 2010	
3. TITLE (The complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S, C or U) in parentheses after the title.) Forecasting Attrition Volume: A Methodological Development			
4. AUTHORS (last name, followed by initials – ranks, titles, etc. not to be used) Fang, M.; Okazawa, S.			
5. DATE OF PUBLICATION (Month and year of publication of document.) December 2009	6a. NO. OF PAGES (Total containing information, including Annexes, Appendices, etc.) 52		6b. NO. OF REFS (Total cited in document.) 12
7. DESCRIPTIVE NOTES (The category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.) Technical Memorandum			
8. SPONSORING ACTIVITY (The name of the department project office or laboratory sponsoring the research and development – include address.) DGMPRA 101 Colonel By Drive Ottawa, Ontario K1A 0K2			
9a. PROJECT OR GRANT NO. (If appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant.)		9b. CONTRACT NO. (If appropriate, the applicable number under which the document was written.)	
10a. ORIGINATOR'S DOCUMENT NUMBER (The official document number by which the document is identified by the originating activity. This number must be unique to this document.) DGMPRA TM 2009-025		10b. OTHER DOCUMENT NO(s). (Any other numbers which may be assigned this document either by the originator or by the sponsor.)	
11. DOCUMENT AVAILABILITY (Any limitations on further dissemination of the document, other than those imposed by security classification.) Unlimited			
12. DOCUMENT ANNOUNCEMENT (Any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). However, where further distribution (beyond the audience specified in (11) is possible, a wider announcement audience may be selected.))			

13. ABSTRACT (A brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), (R), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual.)

Accurate attrition forecasting is crucial for properly planning the recruitment and training of Canadian Forces (CF) members and maintaining the CF strength as well as for managing the CF budget.

This report documents a methodological development in forecasting attrition volume; new procedures for forecasting attrition based on years of service (YOS) have been proposed. The report provides a discussion regarding the rational for the new procedures and explanations why the predictions based on new procedures better reflect CF attrition behaviour. In the end, the new procedures were validated and compared with previous procedures using the real CF personnel data.

The results from the new procedures showed a strong agreement between forecast and actual attrition. Compared with the previous forecasting method, the new procedures deliver predictions that better reflect CF attrition behaviour at many levels.

The report recommends using the proposed procedures for forecasting CF attrition. These procedures can be applied to attrition analyses at many levels within the CF, for example, non-commissioned members (NCMs) and officers (OFFs), Army, Navy and Air Force, different military branches and different military occupations, etc.

This report is targeted at the analysts within Defence Research and Development Canada. This work will equip them with a better approach for forecasting CF attrition, and improve the consistency and transparency of attrition analyses across different research groups. Given that forecasting attrition is so important for a number of relevant human resources initiatives, such as effective recruitment, promotion, planning, and budget management, this work will have a positive impact in all of these areas.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (Technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus, e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

Attrition; Attrition Forecast; Attrition Forecasting; Attrition Modelling; Operational Research; Attrition/Retention; Attrition Methodology

DEFENCE **R&D** DÉFENSE

DRDC CORA

www.drdc-rddc.gc.ca